INDUCTIVE MAGNETIC PROBE DIAGNOSTICS IN A PLASMA

by

Terrence Adrien McLaughlin

United States Naval Postgraduate School

THESIS

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Terrence Adrien McLaughlin
Lieutenant, United States Navy
B.S.E.E., Tufts University, 1964

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ABSTRACT

Inductive magnetic probes were studied, designed, constructed, calibrated and used at the Plasma Study Facility. The probes were built for three specific purposes: to measure current, radial shock waves and Alfvén waves in a fast Theta Pinch; to measure the parameters of shock fronts with short rise times produced by laser exploding of metal fibers; and to observe instabilities in a steady state plasma. Theta Pinch current measurements gave a current of 110 KAmps with the Theta Pinch operating with the capacitors charged to 13KV of 25KV maximum. The ringing frequency of the Theta Pinch was measured as 179 KHz. On calibration, the shock front probes showed a first resonance at 40 MHz which is comparable to probe characteristics found in recent literature. The instability probes gave information which agreed with previous descriptions of instabilities based on optical and Langmuir probe diagnostics.

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I. INTRODUCTION

The determination of the characteristics of transient, shocked or unstable plasmas often requires a detailed knowledge of the time dependence and spacial dependence of magnetic fields. Such information yields important currents in the plasma and along with other diagnostic information gives a complete description of the phenomena under investigation.

The well proven method of obtaining direct magnetic field information from a plasma is by the use of magnetic probes. A magnetic probe is simply an inductive coil arranged to couple with the time changing magnetic field, which gives an electrical output proportional to the time rate of change of the field.

The work undertaken in this thesis was to study, design, construct, calibrate and use inductive magnetic probes in order to add another diagnostic tool for the continuing study of collisionless shock waves at the Plasma Study Facility.

Concurrent with this project were projects to produce shock waves by a Theta Pinch and also by laser exploding of metal fibers. The construction phases of these two projects were completed as this thesis was written. The hardware and techniques developed in this thesis will be used in the determination of the parameters of the shock fronts. The probes coupled with the optical, electrostatic probe and laser scattering diagnostics will give rather complete conventional means of getting information about the shock waves.

The probes actually constructed were for three purposes: 1) to measure the field strength, risetime, and ringing frequency of the Theta Pinch; 2) to measure the form and absolute magnitudes of the shock waves from the Theta Pinch and laser produced plasma; 3) to investigate the instabilities of the steady state plasma in order to provide some information about the background magnetic disturbance in the plasma. Some quantitative information was obtained with the instability probes and was compared with optical measurements made by Andrews [1].

II. THE PLASMA STUDY FACILITY

The Plasma Study Facility of the Naval Postgraduate School produces a steady state plasma in Nitrogen, Argon or Helium with a hollow cathode reflex arc. The plasma beam is contained in a nine foot long pyrex vacuum chamber four inches in diameter. The pyrex chamber is surrounded by six coils which produce an axial magnetic field variable up to 10,000 gauss and homogeneous to within 2.5% along the axis of the plasma beam. The pyrex tube has access ports between the magnets on each side of the tube which are available for access to the plasma beam. The Plasma Study Facility is arranged as shown in Fig. 1.

An example of some typical operating conditions for the Plasma Study Facility are: a neutral gas pressure of 4×10^{-4} torr, a cathode-to-anode current of 50 amperes, nitrogen gas, and an axial magnetic field of 2280 gauss. Under these

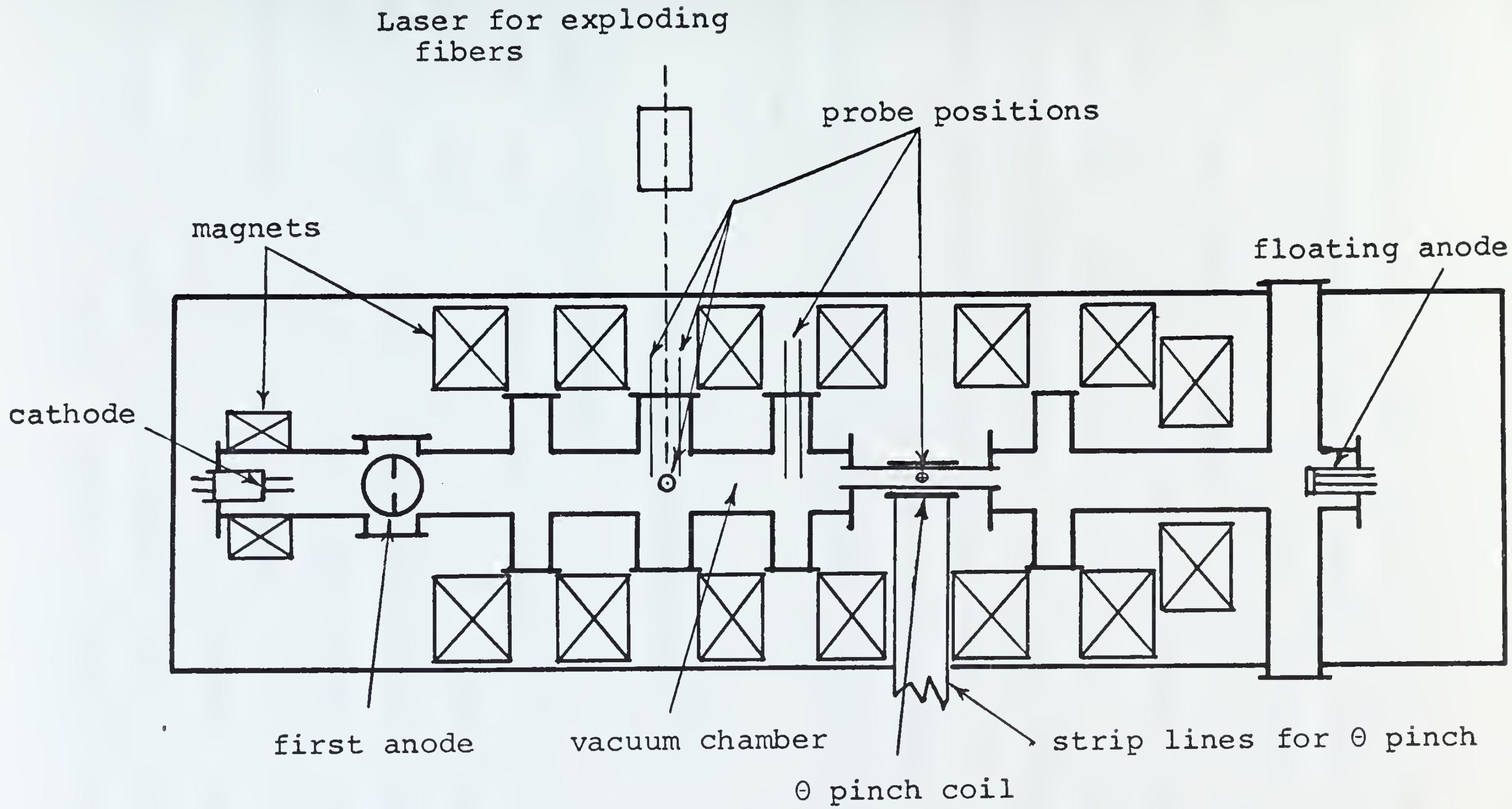


Figure 1. Plasma Machine

conditions the electron temperature on the axis of the plasma beam is 4ev and the electron density is 10^{13} cm^{-3}. For nitrogen, the ion density would be about 5×10^{12} cm^{-3} since $n_e = 2n_i$ [1].

The methods for producing shock waves are shown in Fig. 1. The Theta Pinch operates by passing a short intense pulse of current through a single turn coil surrounding the vacuum chamber. The rapidly increasing field drives a shock wave radially inward compressing the steady state plasma and existing field. Alfvén waves which travel axially on the plasma beam are also generated. The other apparatus for generating shock waves is the giant pulse laser. By exploding a metal fiber with the laser a hot dense plasma is formed which expands rapidly and generates a shock wave in the steady state plasma.

III. INDUCTIVE MAGNETIC PROBES

A. GENERAL TYPES OF PROBES

There are two general ways to make direct measurements on magnetic fields in a plasma. One is by the use of probes, either inductive probes or magnetogalvanic (Hall effect) probes. The other way is with spectroscopic measurements, either Zeeman splitting or Faraday rotation [2].

The spectroscopic measurements give the mean value of the field along a straight line rather than at a given point as is the case with probes. However, probes do perturb the

plasma. Inductive magnetic probes are inexpensive to make, are more sensitive to fast changes in the field strength, and may be made smaller than Hall probes. It was mainly for the first reason that inductive probes were built for the Plasma Study Facility. With some investigation in current semiconductor technology the Hall effect probe could possibly be decreased in size and improved in sensitivity.

B. BASIC CONCEPTS AND ELEMENTS OF DESIGN

1. Output and Size

The concept involved in the operation of an inductive probe is rather simple. The probe consists of a coil of wire placed at the tip of an insulating tube containing other elements for support, shielding and conduction of the signal from the coil. The physical orientation and electrical parameters of the coil are determined by a calibration procedure. The probe is inserted in the time varying field to be measured, usually through a vacuum seal that allows movement of the probe for exact positioning in a vacuum chamber. The electrical output of the probe is proportional to the time derivative of the magnetic field linking the coil. According to electromagnetic theory, the output of a coil is

$$e = nA\frac{dB}{dt} \qquad (1)$$

Where
- n = number of turns of wire on the coil
- A = cross sectional area of the coil
- B = magnetic field
- e = emf generated across the coil.

The quantity nA is usually called the "Effective area" of the coil. The signal is then processed in a fashion depending on the phenomenon under investigation and the type of signal coming from the probe. In this section equivalent circuits, frequency response and shielding will be discussed. Following sections give construction and specific signal processing circuits.

The output of the probe is proportional to nA and the inductance of the probe is proportional to n^2r, where r is the radius of the coil and n the number of turns on the coil. A large output is desired for good signal-to-noise ratio while low inductance is desirable for high frequency response. A figure of merit can be developed by taking the ratio $\frac{nA}{n^2 r} \sim \frac{r}{n}$ which will be the largest for the largest output with the smallest inductance. The good coil would then have a large radius with a small number of turns [3]. In order to achieve minimum perturbing effects on the plasma the probe must be as small as possible. Generally probes in the size range 1mm to 3mm outside diameter are used. Another reason for such small size is the requirement for mapping fields. The fluctuating field should be measured in a region small compared with the scale of phenomena being investigated so that the spacial dependence of the field can be determined.

2. Frequency Response and Shielding

Equation (1) describes an ideal coil with no resistance or capacitance, and connected to an infinite resistance. If the frequencies of interest are relatively low (i.e., the

reactive elements are not important compared to the resistive elements) and the probe coil is connected to a 50 ohm transmission line, then an equivalent circuit for the probe and transmission line will be as shown in Fig. 2a.

To establish what a "low" frequency might be for a small probe, consider a 10 turn coil of copper wire with coil dimensions of .5mm radius and 1mm length. The resistance of the wire will be much less than r_o (resistance of the terminated transmission line) and may be neglected. At low frequencies the reactive impedance of the coil (ωL_p) will be much less than r_o. For purposes of an estimate use $\omega L_p = r_o/10$. The inductance of the coil is $L_p = Fn^2 r \times 10^{-9}$H where F is a form factor depending on the dimensions of the coil (for $r/l = .5/1 = 1/2$, $F = 13$) and where r is in cm. The form factor F may be found in reference [2] or any electrical engineering handbook.

$$L_p = Fn^2 r = 13.0(10^2)(.05)(10)^{-9} \text{ H} = .065\ \mu\text{ H}$$

$$\omega L_p = r_o/10$$

yield $f = \dfrac{r_o}{2\pi L_p 10} = 12.2\ (10)^6$Hz for a 50Ω terminated transmission line.

This estimate shows that the "relatively low" frequencies for such small probes are really rather high.

Other factors also enter. As the frequency is increased the coil capacitance must be included in the equivalent circuit thus leading to Fig. 2b, the equivalent circuit used by Segre and Allen to describe the characteristics of probes up to 30 MHz [4]. Additional capacitance is present if the

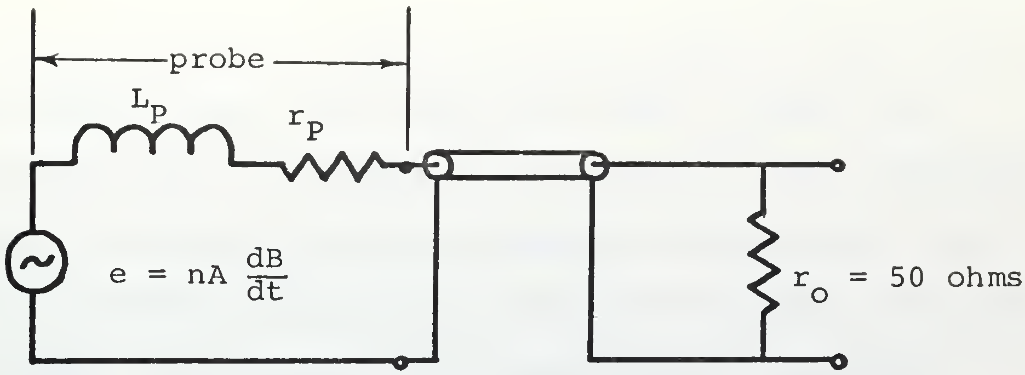
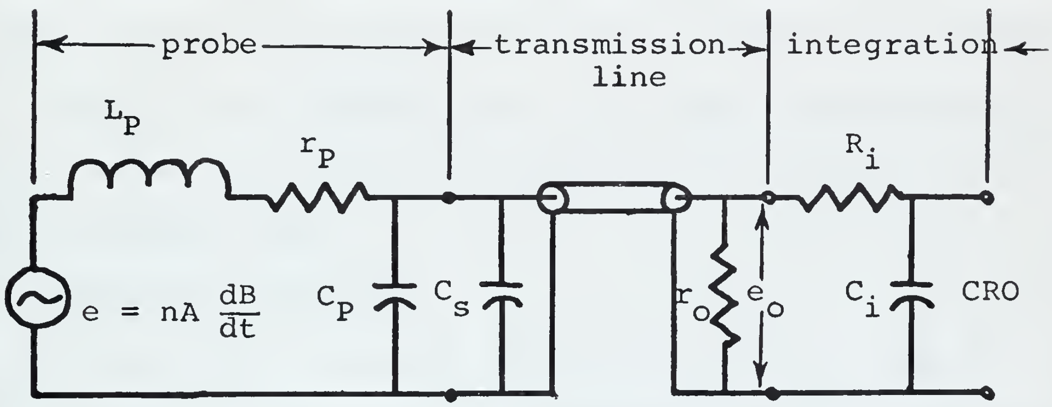


Figure 2a. Simplified Equivalent Circuit for Magnetic Probes at Low Frequencies



$$\left| e/e_o \right| = \left[(r_o + r_p)^2 / r_o^2 + \Omega^4 - \Omega^2 (2 - r_p^2/R_o^2 - R_o^2/r_o) \right]^{-\frac{1}{2}}$$

where: $R_o = \sqrt{L/C}$ and $\Omega = \omega\sqrt{LC}$

L_p = inductance of probe

C_p = capacitance of probe

r_p = resistance of probe

C_s = stray capacitance

r_o = transmission line terminating resistance

R_i = integrator resistor

C_i = integrator capacitor

Figure 2b. Equivalent Circuit for Probe and Associated Equipment at High Frequencies.

coil is provided with electrostatic shielding in the form of a thin metal or mesh cage around the coil. These stray capacitances form a resonant system with the coil inductance, and thus degrade the frequency response of the probe. An unshielded probe will raise the resonant frequency, but then the designer is concerned with electrostatic coupling (capacitive coupling), which is definitely undesirable. Fortunately, it has been shown experimentally that by using a terminated transmission line of low characteristic impedance (50 to 200 ohms) the electrostatic signal is made so small that shielding of the probe coil itself becomes unnecessary [2,3]. Thus by not shielding the coil itself some frequency degrading stray capacitance is avoided while electrostatic pickup is usually small.

3. Transient Response

If the probe is to be used for transient response then transient analysis must be performed. Such an analysis was made by Ferrari and Zucker [5]. From their analysis they present a convenient rule of thumb for design: at a frequency equal to 1/10 of the resonant frequency of the probe, the maximum distortion expected is about 10%. This puts a very difficult restriction on probe design. For example, if the transient phenomenon under investigation has a waveform with a fundamental frequency of 250 KHz, and if 5 harmonics with less than 10% distortion are desired, then the probe must show no resonance below 12.5 MHz. Furthermore there

should be no significant frequency components in the transient signal higher than the 5th harmonic (12.5 MHz).

It may be convenient to make simultaneous measurements of the magnetic field at a number of positions in a plasma. This can be done by including two or more coils in one probe jacket to measure either the same component of field at different positions or different components of field in adjacent positions. Many investigators have used multicoil probes and have found negligible interference between the signals from separate coils [6,7].

C. CONSTRUCTION TECHNIQUES

The mechanical construction of the probes for the Plasma Study Facility was similar to that used by other investigators, with a few modifications to achieve higher frequency response.

In general a probe consists of an outer insulating jacket to provide protection from the hot plasma and to insulate the probe from the plasma; a support tube, usually conducting so as to provide shielding; a pair of lead wires to carry the signal from the coil; the coil itself wound on a form or free standing; and sometimes an electrostatic shield around the coil.

Pyrex glass was used as the outer jacket on all the probes. One pyrex probe resisted the steady state nitrogen plasma under reasonable operating conditions for about ten hours of data taking. A second pyrex probe was destroyed

near the center of the beam within five minutes after the nitrogen input pressure to the hollow cathode arc was increased by a factor of three. If further steady state measurements were to be made near the beam, it would be advisable to use quartz or alumina tubing for the outer jacket. Probes other than the instability probes will not be used in the center of the steady state plasma, and will be used to measure fast transient effects. Therefore, pyrex tubing will suffice for the foreseeable measurements.

Coaxial cable was used exclusively for internal wiring in order to prevent as many electrical missmatch situations as possible and to use the shielding inherent in coaxial cable. A conducting support tube was used in the pinch and waveform probes in order to provide a second shield as close as possible to the actual coil position. The Theta Pinch uses large spark gaps and will be a copious source of interference so the secondary shield will offer some relief from radio frequency noise.

The shockwave probe was the most exacting probe built, since it was recognized early that it would be providing the data of greatest importance. The actual coil is a free standing single layer coil .6mm by 1.2mm wound from #40 Formvar insulated wire. It is soldered directly to a solid jacketed microcoaxial cable, made rigid with an epoxy cement, and potted with a silicone rubber compound in a 1mm outside diameter pyrex tube. The 1mm tube is about 5cm long and

then joins a 5mm tube. The coaxial cable is supported by nylon spacers in a supporting stainless steel tube. The supporting tube holds the fixture for a miniature connector for RG174 coaxial cable. All parts are impedance matched at 50 ohms. Constructing such a small probe is no easy task and must be done by an experienced technician. Figures 3 and 8 show other details of the three types of probes constructed.

D. CALIBRATION TECHNIQUES

1. General

The more desirable features of the inductive probe are that it can be calibrated very accurately and that its response is linear and stable. There are a variety of ways of calibrating an inductive probe. The most widely used, with many variations, is with the use of sets of Helmholtz coils. This method has been used successfully up to 50 MHz by Phillips and Turner [6]. Helmholtz coils offer an easily calculated and nearly homogeneous field in an appreciable area between the coils. Thus they are well suited for probe calibration. Helmholtz coils similar to the design of Phillips and Turner were built and used. The design is shown in Fig. 4. The Helmholtz coils were used in two ways based on the following equations:

$$B = \frac{N\mu_o i}{r}\,\frac{8}{5^{3/2}}$$

where

B = magnetic field on the axis midway between the Helmholtz coils

i = current in the Helmholtz coils

N = number of turns on each coil

r = radius of Helmholtz coil

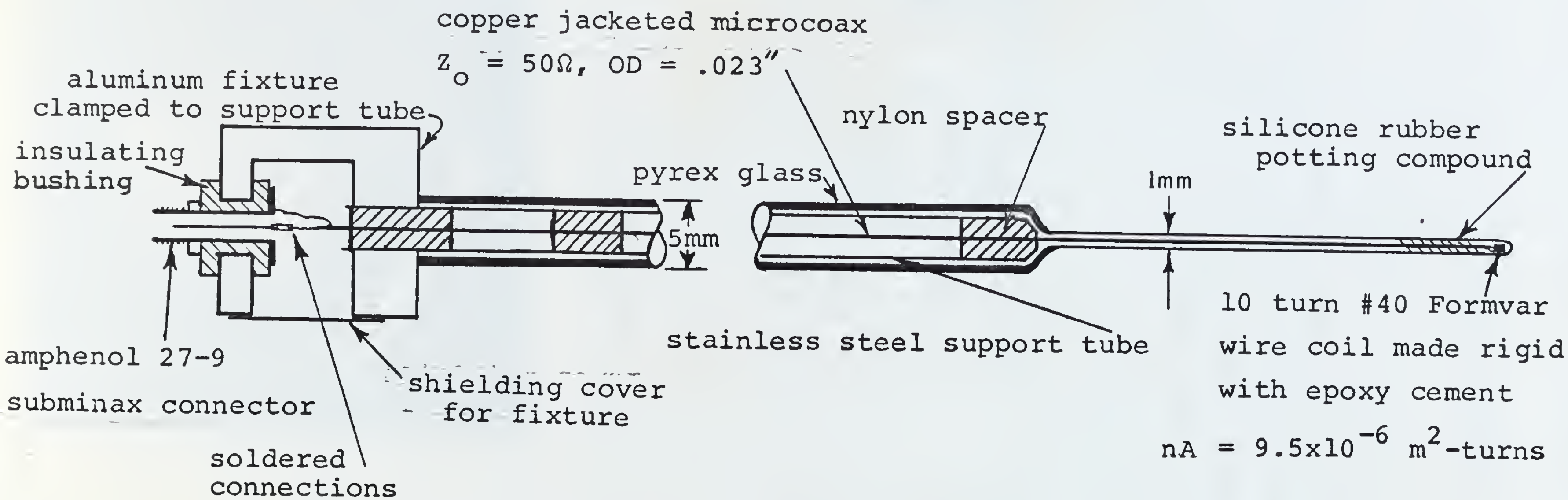
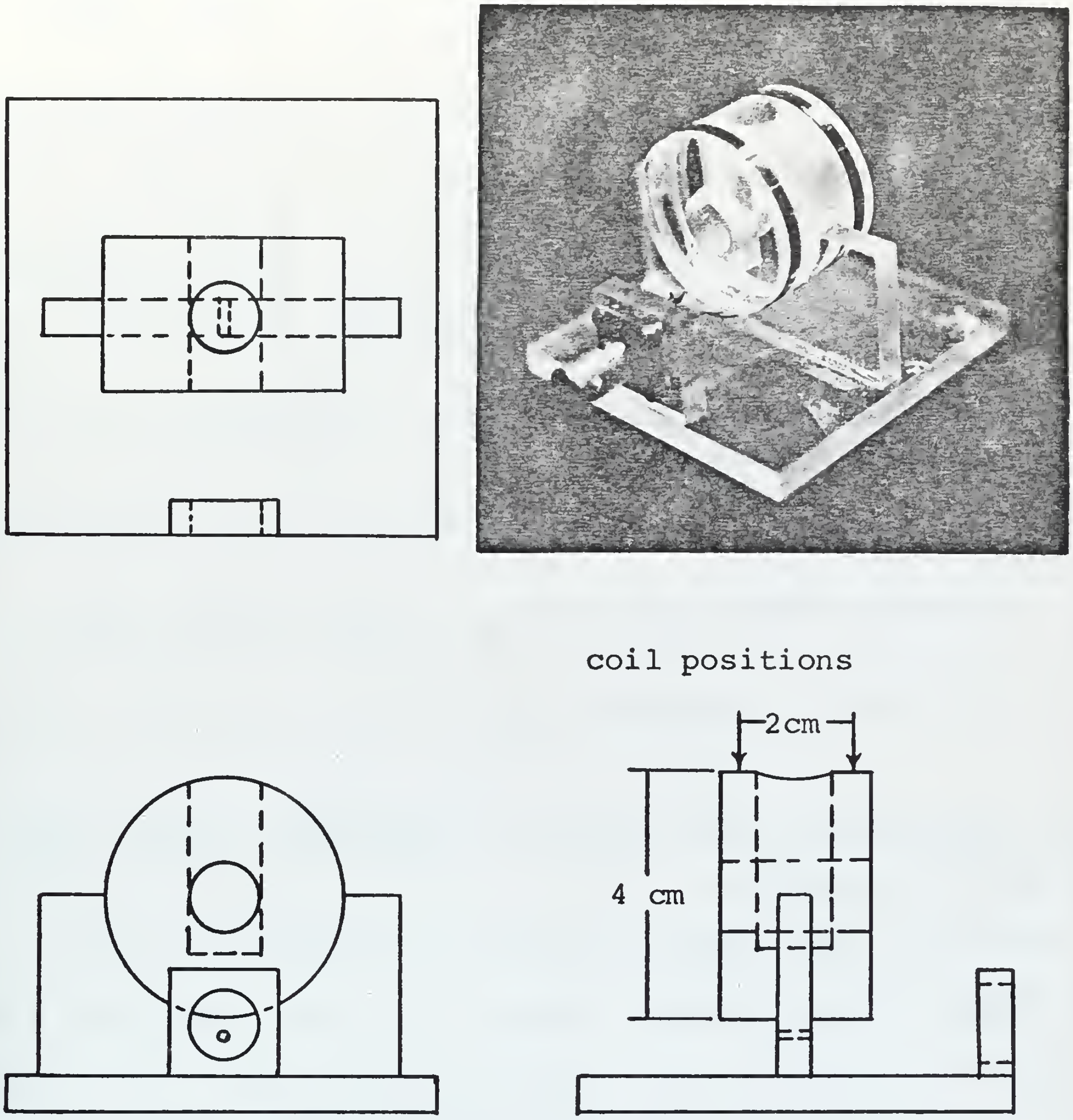


Figure 3. High-Frequency Probe for Shock Wave Measurements.



scale 3/4" = 1"

Materials: lucite for coil forms
#28 enameled copper wire for coils
Panel jack for RG 58 coaxial cable
four sets made with N(number of turns on each coil) = 1,2,5,10

Figure 4. Helmholtz Coils.

$e = nA \frac{dB}{dt}$ where nA = effective area of the probe coil
e = voltage output from probe coil.

A sinusoidal generator provided the driving current for the Helmholtz coils so we can write $i = I\sin\omega t$, $e = E\sin\omega t$, from which we obtain $e = nA \frac{N\mu_o 8}{r\, 5^{3/2}} \omega I \cos \omega t$

since RMS values are measured

$$(2) \quad nA = \frac{E}{fI} \frac{r\, 5^{3/2}}{2\pi n \mu_o 8} = g \frac{E}{fI}$$ where g is constant for a given set of coils.

Another form of the equation can be formed by using

$v_L = L \frac{di}{dt}$ where v_L = voltage across Helmholtz coils
L = inductance of Helmholtz coils.

$$(3) \quad \text{thus } nA = \frac{EL}{V_L} \frac{r\, 5^{3/2}}{8\, N\mu_o} = K \frac{E}{V_L}$$ where K is constant for a given set of coils.

At low frequencies nA will represent the true turns - area of the probe coil. At higher frequencies the reactive elements of the equivalent circuits (Fig. 2b) will come into play and add frequency-dependence to the system of probe, transmission line, termination and perhaps integration circuit. Thus, a plot of nA versus frequency shows natural resonant frequencies of the system. Hopefully these resonances are well above the natural frequencies of the phenomena to be measured.

In order to use equation (3) for calibration, values of inductance for the four sets of Helmholtz coils had to be measured. A simple but effective method was used to determine

the Helmholtz coils' inductance. A .0540μf capacitor was charged to 100 volts and discharged across the Helmholtz coils and a polaroid picture was taken of the oscilloscope trace of the voltage across the coils. A typical under-damped resonance was observed. From the frequency of the ringing circuit the inductance was calculated for each set of coils. The results are tabulated in Table 1 along with numerical values of the groups of constants, k and g, appearing in equations (2) and (3).

One set of Helmholtz coils was connected to a pulse generator and the resultant voltage across the coils was observed on an oscilloscope. The ringing frequency of the coils and stray capacitance was approximately half of that reported by Phillips and Turner as the resonant frequency of the coils. The reason is that the output capacitance of the pulse generator, the input capacitance of the oscilloscope, and the capacitance of the connecting lines were all in parallel, and lowered the resulting ringing frequency. Therefore, the Helmholtz coils must always be used with care. By connecting the coils directly to the signal generator and using radio frequency meters as described in the following paragraphs, the calibration coils functioned well to 50 MHz, the upper limit of the signal generator.

The three types of probes to be calibrated naturally separated into three frequency ranges. These will be discussed starting with the lowest frequencies, which are the

TABLE 1

CHARACTERISTICS OF HELMHOLTZ COILS
and
CONSTANTS FOR CALIBRATION EQUATIONS

N Number of Turns on Each Coil	L Inductance of Helmholtz Pair	k	g
10	$L_{10} = 19.04\ \mu H$	$k_{10} = 0.042$	$g_{10} = 356$
5	$L_5 = 5.69\ \mu H$	$k_5 = 0.025$	$g_5 = 712$
2	$L_2 = 1.19\ \mu H$	$k_2 = 0.013$	$g_2 = 1780$
1	$L_1 = .42\ \mu H$	$k_1 = 0.009$	$g_1 = 3560$

$$nA = k\frac{E}{V_2} = g\frac{E}{fI}$$

easiest, and working to the highest frequencies actually used. Then methods for even higher frequency measurements will be suggested.

2. Low Frequency Techniques

Andrews found instabilities in the steady state plasma to be rotational in nature with characteristic frequencies in the 10KHz to the 500 KHz range (1). This frequency range corresponds to what was expected as the ringing frequency of the Theta Pinch (later measured as 179 KHz). Therefore, instability-measuring probes were built and calibrated to determine the magnitude of magnetic fluctuations associated with the instabilities and to confirm the general nature of Andrews' work. The relatively low frequencies involved and the availability of a Hewlett-Packard 3590A wave analyzer made the calibration relatively simple. The basic circuit is shown in Fig. 5a. The wave analyzer has a BFO output which can be considered as a constant amplitude generator with a 600 ohm internal impedance. For $V_L << V_o$ it can easily be shown that $V_L = \frac{\omega L}{r_o} V_o$ where the quantities are as shown in Fig. 5a. V_o was set at one volt so that $V_L/L = \omega/r_o$. Using this in equation (3)

$$nA = \frac{E}{\omega} \frac{r\ r_o\ 5^{3/2}}{N\ \mu_o\ 8} = \frac{E}{f} \frac{r\ r_o\ 5^{3/2}}{N\ \mu_o\ 16\pi}$$

The output of the wave analyzer can be plotted on an XY recorder as E vs f. If there are no resonances, the resulting plot is a straight line passing through the origin. The

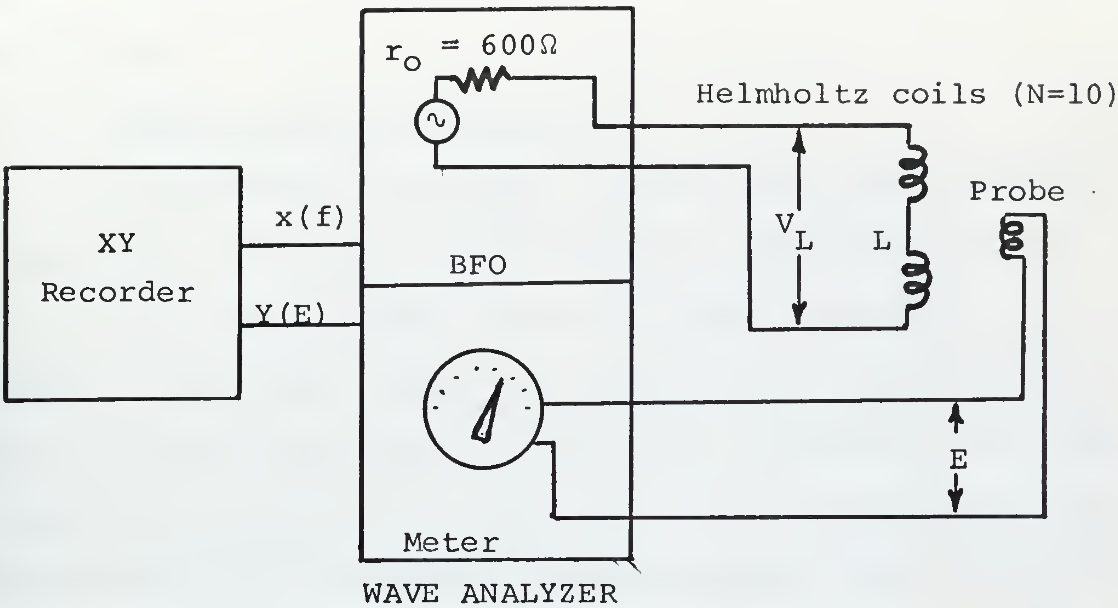
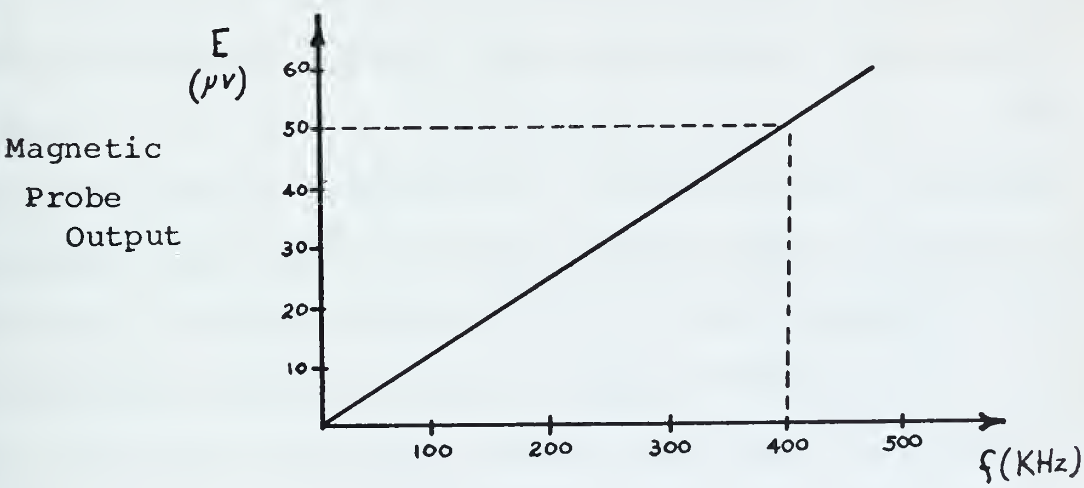


Figure 5a. Low Frequency Calibration Technique



Plot from XY recorder for first instability Probe

$$nA = \frac{r\, r_o\, 5^{3/2}}{N\, \mu_o\, 16\pi}\,\frac{E}{f} = \frac{(.02)(600)(11.2)}{(10)(4\pi)(10)^{-7}(16\pi)}\,\frac{E}{f} = 2.13(10)^{5}\,\frac{E}{f}$$

$$= 26.7(10)^{-6}\ m^2$$

Figure 5b. Low Frequency Calibration Calculation

slope is E/f and nA is thus determined. The instability probes were calibrated in this way. Figure 5b depicts the calibrations.

3. High Frequency Techniques

As discussed previously a Theta Pinch probe capable of measuring a transient waveform with a fundamental frequency in the order of 250 KHz has to show a constant nA response to 12.5 MHz. Likewise, a probe for observing waveforms of shock fronts has to have a flat response to as high a frequency as practical. With two slight modifications the method of Phillips and Turner [6] was adopted for calibrating those probes. The basic circuits are shown in Fig. 6. The basic equations were previously given as equations (2) and (3). The method using V_L and E was easiest to apply and gave very satisfactory results. The calibration of the pinch probes to 12.5 MHz was accomplished without using a screen room. For higher frequencies a screen room was necessary. Equipment consisted of a General Radio 1330A continuous wave oscillator, Hewlett-Packard 411A r.f. VTVM, Helmholtz coils as described previously, and a Simpson VTVM with r.f. probe. The calibration curve for a shock front wave form probe is shown in Fig. 7. The calibration curve of a Theta Pinch probe to 12.5 MHz consists of a horizontal straight line to 12.5 MHz.

During the calibration process it was convenient to indicate for future reference the orientation of the coil.

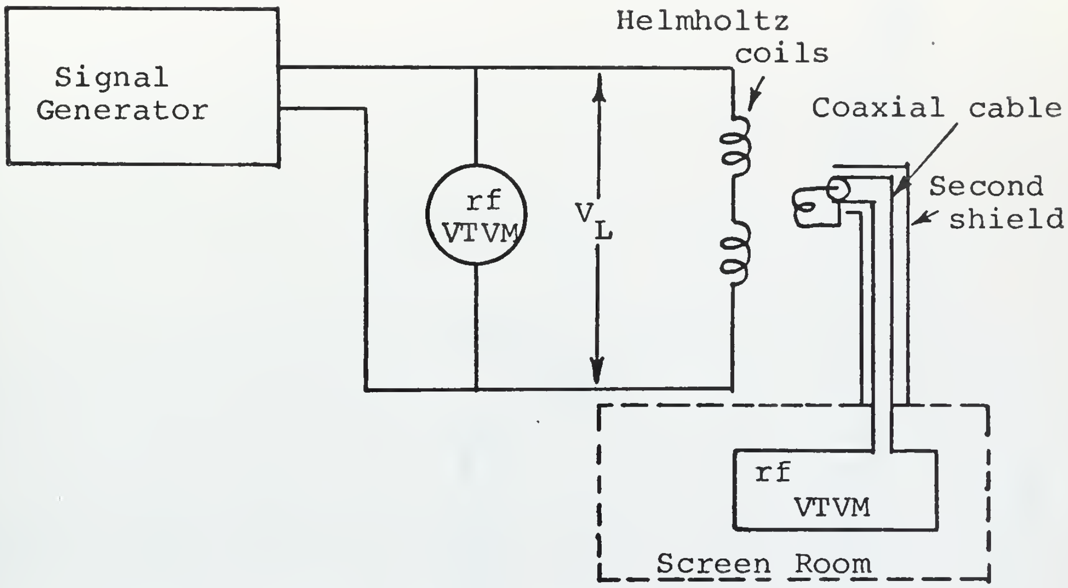


Figure 6a. Method for High Frequency Probe Calibration

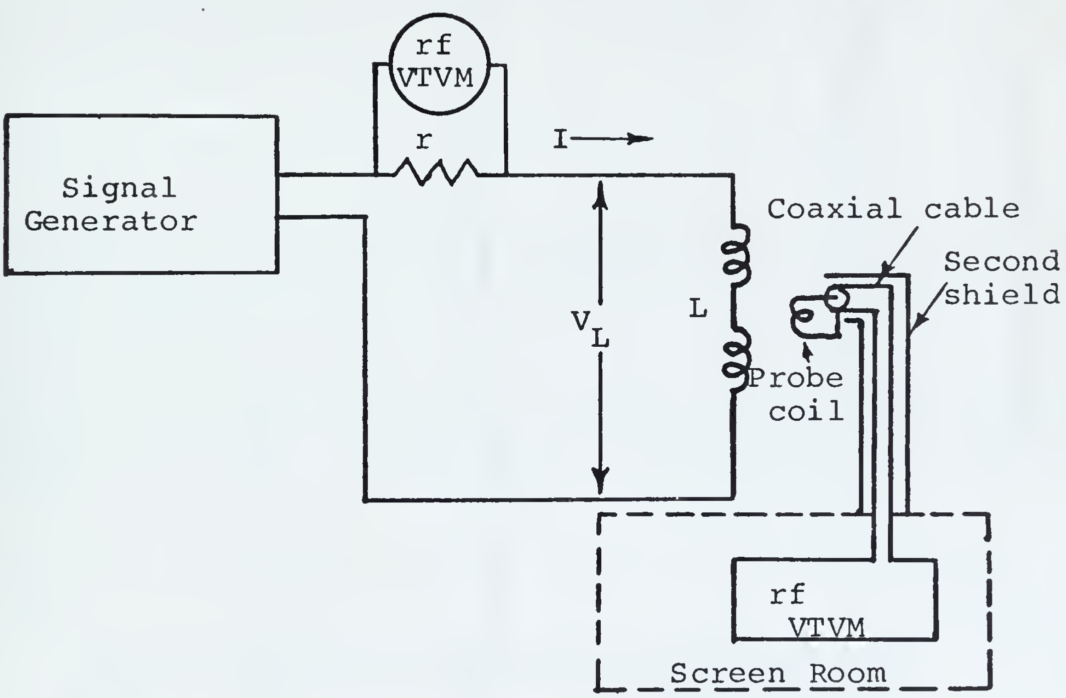


Figure 6b. Methods for High Frequency Probe Calibration

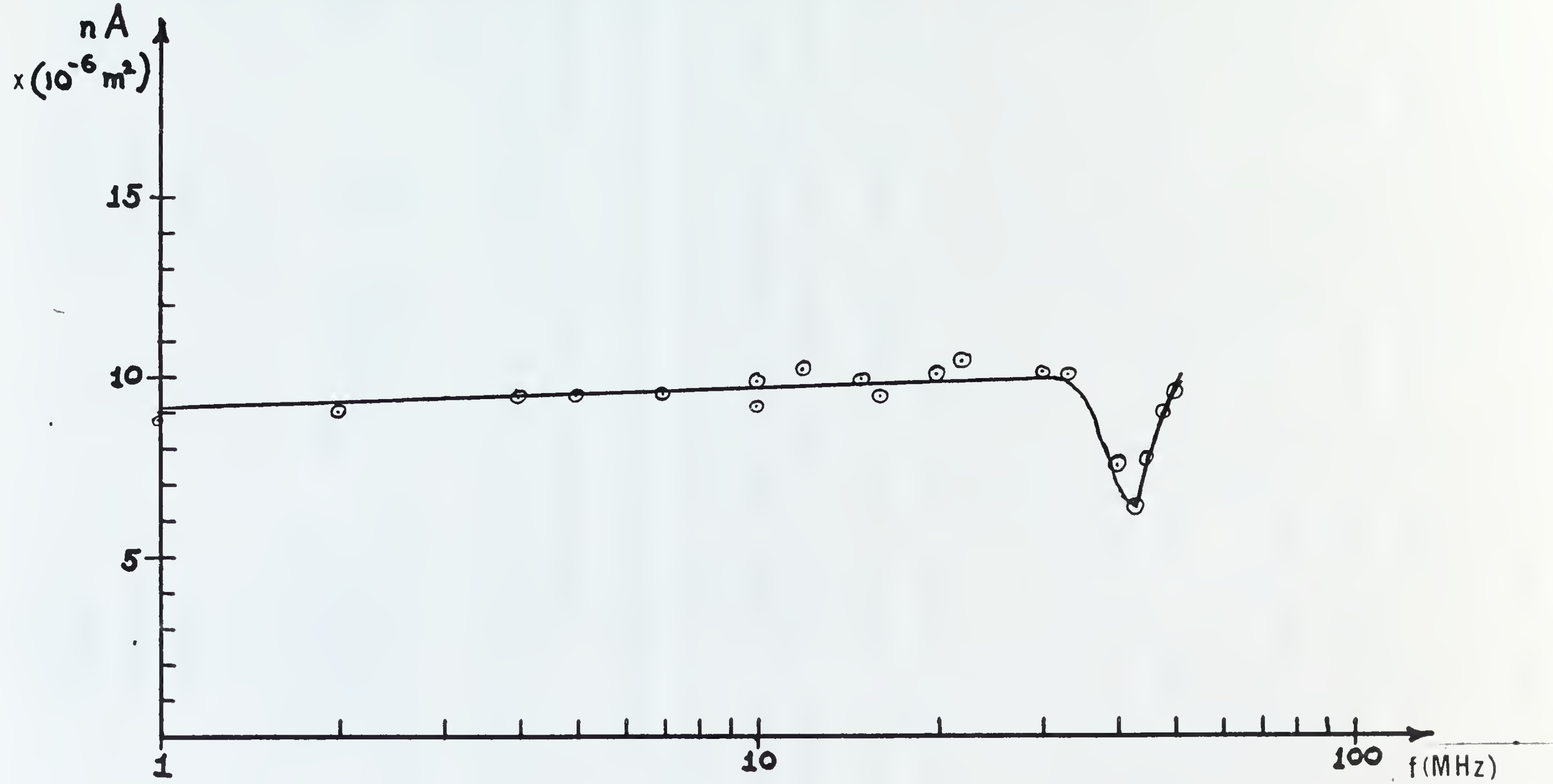


Figure 7. Calibration Curve for a Shockwave Probe

This was done by cementing a small lucite pointer to the end of the probe so that the axis of the marker was parallel to the plane of the coil itself. The plane of the coil was easily determined by rotating the probe in the center of the Helmholtz coils until a maximum voltage output was obtained from the probe. The plane of the probe coil was then parallel to the plane of the Helmholtz coils. Small dials were also made to fit behind the pointers in such a manner as to rotate independently of the pointer. By fixing the dial, a reference plane can be established and the plane of the probe coil can be rotated with respect to that plane.

Other calibration techniques may be found in the literature. Ringing circuits, transistorized circuits and large coaxial lines have been used [2,7,8]. The use of a large coaxial transmission line much like the slotted lines used in microwave equipment offers a good way to get a sinusoidal magnetic field of very high frequency.

E. CHARACTERISTICS OF PROBES CONSTRUCTED

The characteristics of the probes constructed are tabulated below.

Type of Probe (Purpose)	nA (m^2)	Outside Diameter at Tip (mm)	First Resonance Frequency in Operating Configuration	Number Constructed
Shockwave	9.8×10^{-6}	1	33 MHz	3
θ Pinch	1.6×10^{-6}	1	12.5+MHz	1
	2.9×10^{-6}	2	12.5+MHz	1
Instability	26.7×10^{-6}	3	.5+MHz	1
	29.1×10^{-6}	3	.5+MHz	1

All probes were calibrated in an operating condition, i.e., the measuring instruments were located remotely from the plasma machine and enough coaxial cable was connected to reach the plasma machine. Calibrating the probe and cable as a system circumvents any calculations of attenuation in the transmission lines. The RG174 connecting cables used in the laboratory are all 35 feet in length. Cable for use inside the screen room is RG58 and each cable is 8 feet in length.

IV. TECHNIQUES AND EXPERIMENTAL RESULTS

A. GENERAL

The measurements are made with a complete system consisting of the probe, a connecting cable to the screen room feed-through, and the recording device inside the screen room.

The construction and calibration of the probes has been discussed. The positioning, shielding and recording devices will now be considered for the various types of probes. Measurements were made using the Theta Pinch and instability probes. Unfortunately, while the construction phases of the shock production devices were completed, minor difficulties prevented any consistent shock wave production for actual measurements.

B. THETA PINCH CURRENT MEASUREMENTS

The Theta Pinch coil, probe holder, and probe are illustrated in Fig. 8. The measurement from the pointer to the

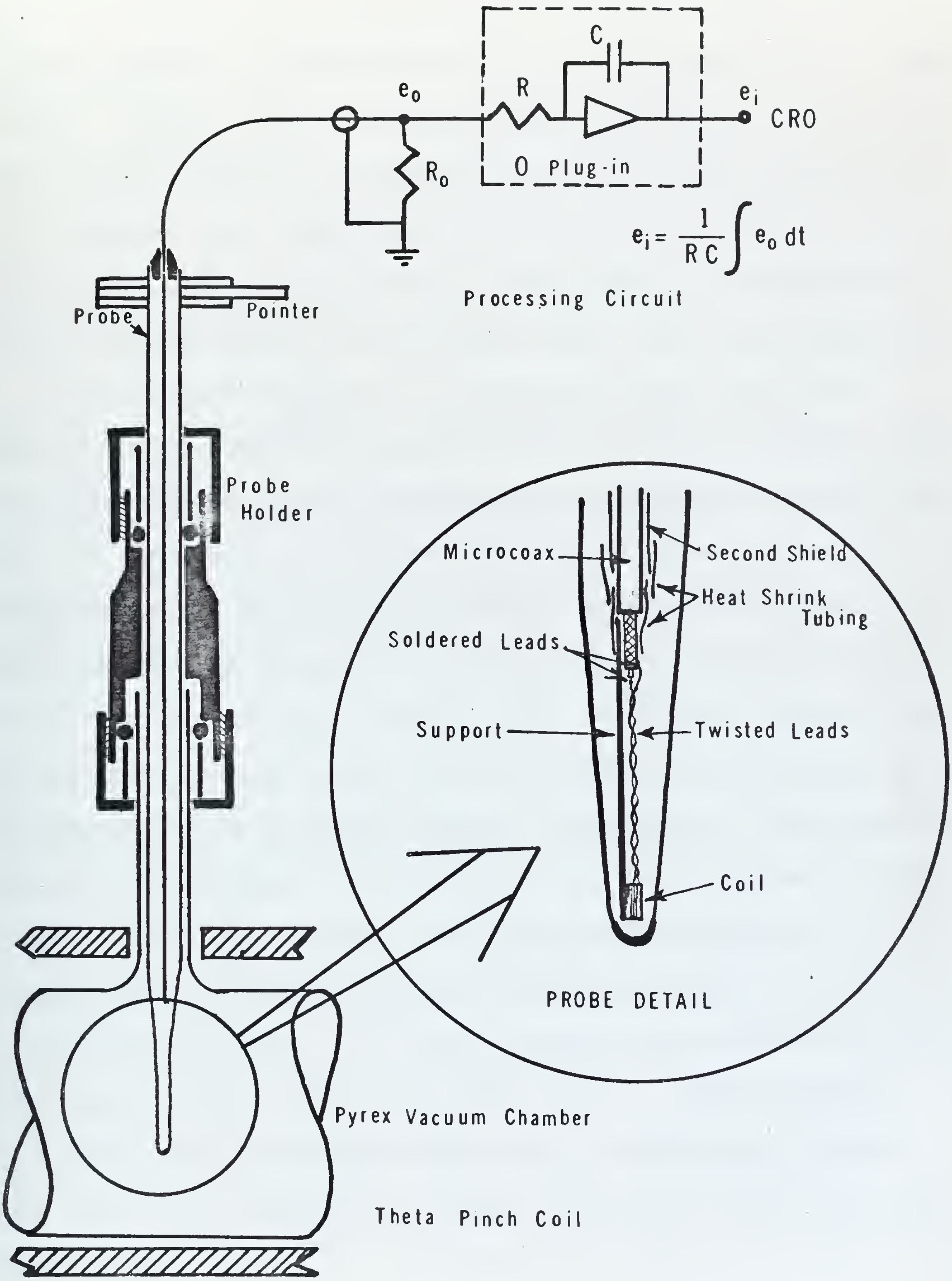


Figure 8. Theta Pinch Probe and Accessories

probe holder gives radial positioning and the pointer installed during calibration gives angular positioning. The second shield of the probe can either be grounded at the plasma machine or connected to a second shield for the whole cable. If the latter is done it is important to insure that the second shield is grounded at only one point; preferably the screen room. The second shield may be either a copper tube or interweave shielding. The connection between the probe and cable must be insulated from the second shield.

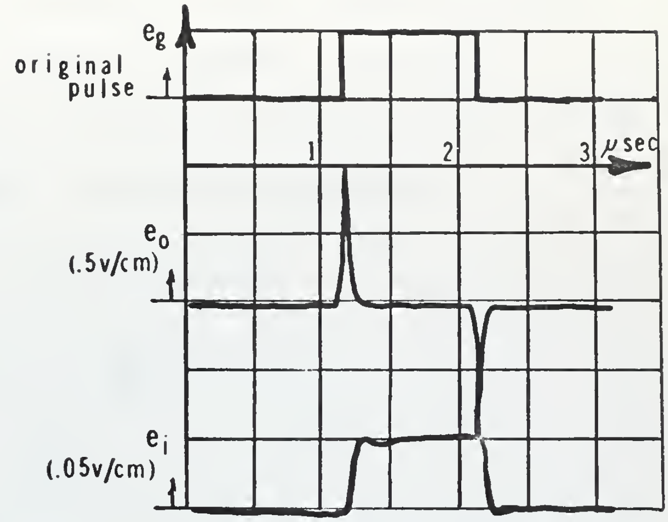
The recording device is a Tektronix 555 dual beam Oscilloscope with "H" type and "O" type plug-in units. The "H" type plug-in gives good frequency response to 30MHz and the "O" type plug-in allows integration to be performed. A diagram of the integrating circuit incorporated in the "O" type plug-in is shown in Fig. 8. Because of the operational amplifier a good first estimate of the RC time constant when using the "O" type plug-in is the period of the signal to be integrated. If a purely passive integration circuit was used the RC time constant would have to be 5 to 10 times the period of the input signal. To test the performance of the integrating circuit, a simple ringing circuit was built, with a ringing frequency of 1.5 MHz using a pulse generator and an RC tank circuit. With $R = .01M\Omega$ and $C = .0001\mu f$ so $RC = 1\mu s$ the signal was integrated with very satisfactory results. The test data along with a pulse integration test are shown in Fig. 9.

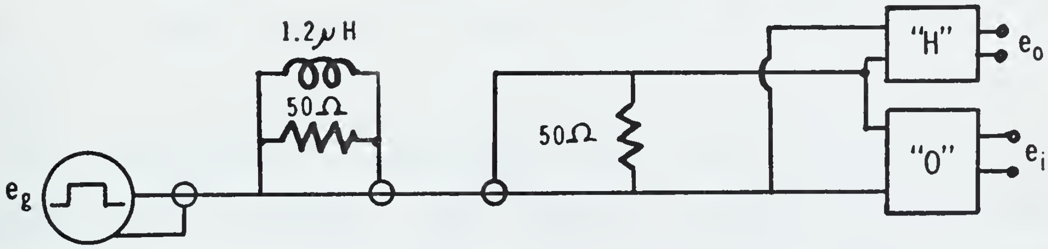
Pulse Integration

Sweep .5μsec/cm

$$e_i = \frac{1}{RC}\int e_o\,dt$$

R = .1M
C = 10pf
RC = 1μsec



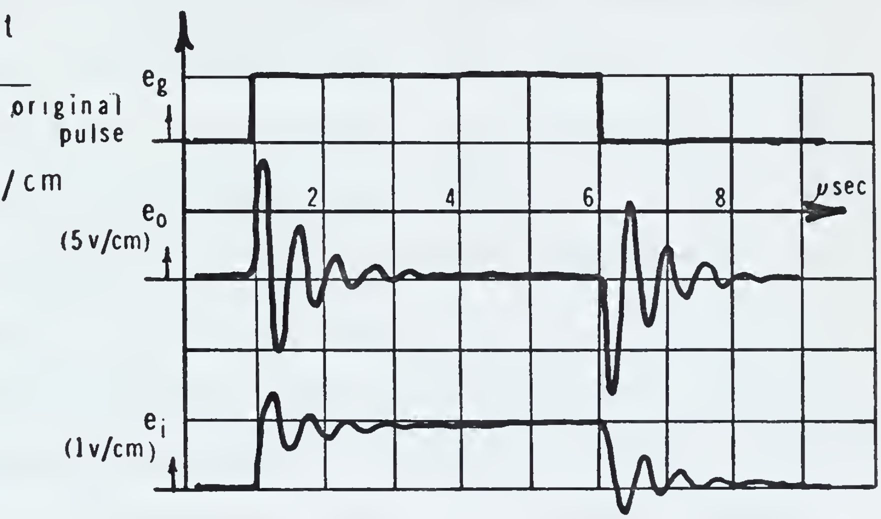


Ringing Circuit Integration

Sweep 1μsec/cm

$$e_i = \frac{1}{RC}\int e_o\,dt$$

R = .01M
C = 100 pf
RC = 1μsec



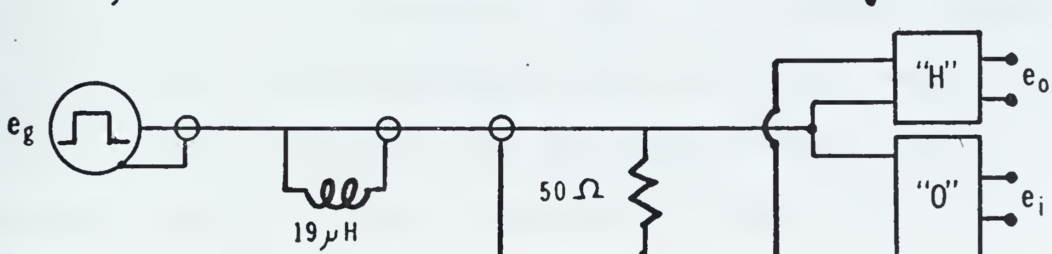


Figure 9. Integration using "O" Plug-in

The integration tends to eliminate the high frequency components of a waveform. This is because the output voltage from the integrator is inversely proportional to the frequency of the input. This can easily be shown:

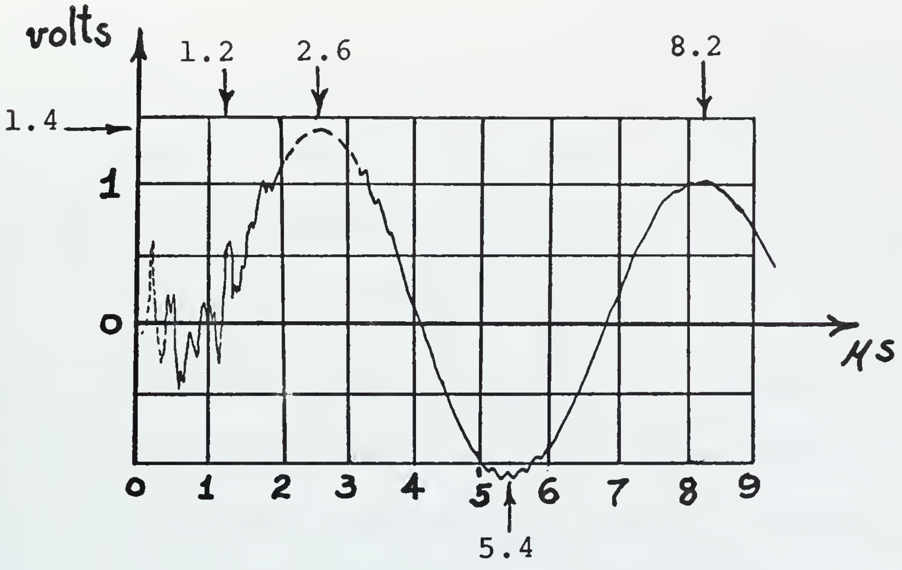
$$e_o = \frac{1}{RC}\int e_i \, dt \text{ for the integrator}$$

$$\text{if } e_i = E \sin \omega t \quad \text{then } e_o = \frac{E}{RC\omega} \cos \omega t$$

$$\text{and the gain } G = \left|\frac{e_o}{e_i}\right| = \frac{1}{RC\omega} .$$

Also, the upper frequency limit of the "O" type plug-in is 13 MHz.

The parameters of the Theta Pinch were estimated by those building the equipment. The ringing current in the Theta Pinch coil was designed ideally to have a maximum of 500 KAmps to 600 KAmps and a ringing frequency of about 200 KHz with the 42μf capacitor bank charged to a full voltage of 25 KV. A risetime of about 1μs was expected. To measure the parameters experimentally, the equipment was set up as described, except that no secondary shield was used. The voltage at the probe was expected to be larger than any radio frequency signal from the spark gaps. Also, the pinch coil itself offers some shielding from radio frequency signals. The first trace observed is reproduced in Fig. 10. Further attempts yielded copious high frequency noise, probably radio frequency, and very little signal in the frequency region expected from the Theta Pinch. An arc in the strip lines close to the pinch coil was suspected.



Tektronix "H" plug-in with vertical at .5 V/cm and horizontal at 1μs/cm

$$E(\text{maximum}) = 1.4 \text{ V}, \quad f(\text{ringing}) = \frac{1}{5.6(10)^{-8}} = 179 \text{ KHz}$$

$$I(\text{maximum}) = \frac{L\ E\ (\text{maximum})}{nA\ \mu_o\ \pi f}\left\{1 - \frac{2a^2}{L^2}\right\}^{-1}$$

where a = 2.75 cm, L = 15 cm

$$I(\text{max}) = \frac{(.15)(1.4)}{2.9(10)^{-6}\ 4\pi(10)^{-7}\ \pi\ 1.79(10)^5}\left\{1 - \frac{2(2.75)^2}{(15)^2}\right\}^{-1}$$

$$= 110 \text{ KAmp}$$

Figure 10. Theta Pinch Current Calculation

The probe coil was positioned in the center of the pinch coil with the axes of the coils parallel. Using

$$B_z = \frac{\mu_o N I}{L}\left\{1 - \frac{2a^2}{L^2}\right\} \quad \text{where}$$

a = radius of solenoid

L = length of solenoid

N = 1 = number of turns on solenoid

I = current in solenoid

for the magnetic field at the center of a solenoid and $e = nA\frac{dB}{dt}$ as the output of the probe it follows that

$$I = \frac{L}{nA\mu_o}\left\{1 - \frac{2a^2}{L^2}\right\}^{-1} \int edt \qquad (4)$$

to get the maximum value of current the integral becomes the area under the first positive part of the waveform. Assuming a nearly sinusoidal waveform the integral becomes

$$\int_0^t edt = E\int_0^t \sin \omega t \; dt = \frac{E}{\omega}\int_0^{\pi} \sin \Theta \; d\,\Theta = \frac{E}{\pi f}$$

$$\text{so } I(\text{maximum}) = \frac{L}{nA\mu_o}\left\{1 - \frac{2a^2}{L^2}\right\}^{-1} \frac{E}{\pi f} \qquad (5)$$

where E = maximum voltage in first half cycle,

f = ringing frequency of Θ pinch wave form.

The calculations for the trace shown yield: f(ringing) = 179 KHz and I(maximum) = 110 KAMps with the Theta Pinch capacitors charged to 13 KV.

It should not be difficult to integrate electronically the signals from the Theta Pinch and get I directly from equation (4). Other than current measurements, the probes are capable of measuring the characteristics of radial shock waves generated by the Theta Pinch in the steady state plasma.

At some point, probably at about .5cm from the center of the beam, the probes will be destroyed by heating as energy is transferred to the probe by particle bombardment. This problem will be discussed in section IV.D on instabilities and noise in the steady state plasma.

C. SHOCK WAVES AND ALFVÉN WAVE MEASUREMENTS

No shock waves were produced during the time of this project. Therefore, this section will be concerned only with experimental techniques for measuring the parameters of shock waves and Alfvén waves.

Basically a magnetic probe observes the interaction of a plasma and a magnetic field. In the case of the Theta Pinch the pulse of current through the Theta Pinch coil produces a rapidly rising axial magnetic field. This field acts as a piston producing a shock wave radially inward. The radial compression can be studied by placing a probe as for Theta Pinch current measurements (Fig. 8), but not at the center of the beam. As the plasma is compressed the dividing line between the plasma and the compressed field will move radially inward and thus pass the tip of the probe. The structure of the interface can thus be studied. Integrating circuits can be used to get B from dB/dt if the loss of high frequency components can be tolerated. This loss of information as discussed in section IV.B may force the use of graphical means of

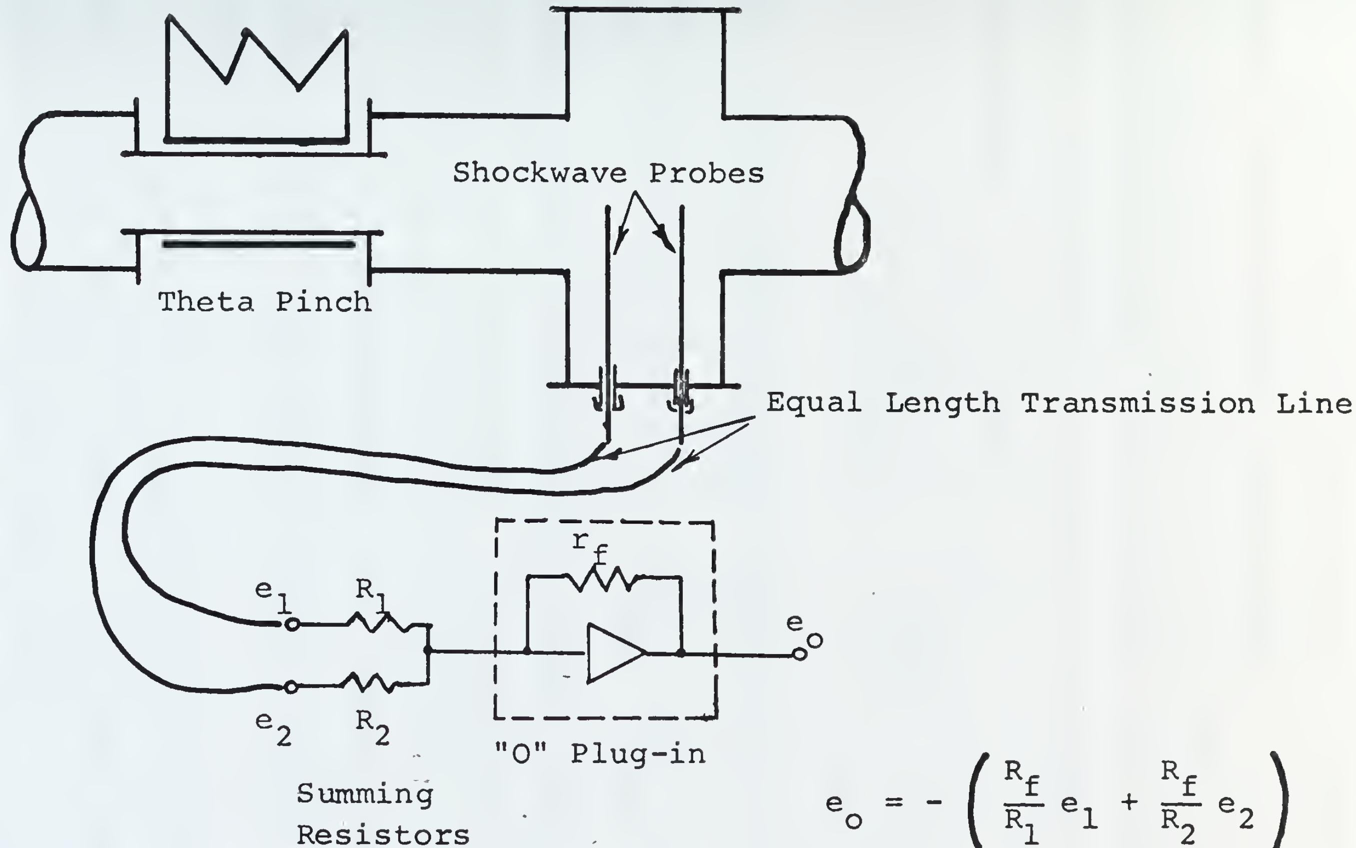
integration. Also, integration with respect to time may not be proper if the plasma is moving with respect to the probe and the probe coil is large compared to the shock wave thickness [9].

The thickness of the shockfront can be estimated from the risetime of the Theta Pinch field and the velocity of the shock front in the plasma. The risetime is approximately 1μ sec and the velocity may be approximated by the Alfvén velocity of 4.5×10^7 cm/sec. The thickness of the shock-front will then be

$$d = v_a t = 4.5(10)^7 (10)^{-6} = 45 \text{ cm}$$

Since this is many orders of magnitude above the 1mm probe coil dimension, no special consideration need be given because of shock front thickness.

Since the pinch is a ringing circuit, an approximately sinusoidal Alfvén wave will be generated and travel axially on the plasma beam. The velocity and attenuation of the Alfvén wave can be studied by placing probes in the nearest port to the Theta Pinch (Fig. 11). By the use of a simple summing circuit (Fig. 11) the outputs of the two probes may be superimposed on the same oscilloscope trace. The distance between the wave forms on the trace will give a time of flight type of measurement. Caution should be taken to insure that all connections for the two probes are equivalent so that differential delays will not be introduced. The attenuation of the Alfvén wave can be measured by comparing the outputs



$$e_o = -\left(\frac{R_f}{R_1} e_1 + \frac{R_f}{R_2} e_2\right)$$

Figure 11. Alfvén Wave Velocity Measurement Circuits

of the two probes. As discussed for the Theta Pinch current measurements, a second shield may be required to shield against radio frequency interference.

Some numerical calculations will show the feasibility of measuring Alfvén wave velocities. The Alfvén velocity along a field line in a plasma in a constant magnetic field of magnitude B_o is given by

$$v_a = \frac{B_o}{\sqrt{\mu_o n_i m_i}} \qquad \text{where } n_i = \text{ion density}, \quad m_i = \text{ion mass}$$

For a steady state nitrogen plasma at the Plasma Study Facility with B_o at $.2 w/m^2$ a typical value for n_i is $5(10)^{18}\ m^{-3}$. A value of v_a is thus $4.5(10)^5$ m/sec. The probes are about 8cm apart so a flight time between probes of .18μ sec. is to be expected. The Tektronix 555 or Tektronix 519 may be used to record the measurement. If two ports are used, the distance between probes can be increased by a factor of 100. The time of flight is then very easy to measure.

The laser-produced plasma will be expanding from the point where a small piece of metal is vaporized and ionized by a laser pulse. A very hot dense plasma is formed with $\beta > 1$. The expanding plasma will push back the static magnetic field produced by the main magnets of the plasma machine. The interface between the plasma and magnetic field can be studied using the shock front probes in much the same manner as for Theta Pinch shock front measurements.

The energy from the giant pulse laser will be deposited in about 10 nanoseconds. The expanding front will have a velocity in the order of 10^7 cm/sec. This leads to a front thickness of $d = (10^7)(10)^{-8} = .1\text{cm} = 1\text{mm}$.
This is on the same order as the dimension of the probe coil so that the entire front will be linked with the coil at the same time. Caution must be used in interpreting any electronically integrated signals from the probe.

D. INSTABILITIES IN THE STEADY STATE PLASMA

Background magnetic fluctuations in the steady state plasma due to instabilities could be a source of interference when making shock wave measurements. Therefore, magnetic probes were built to observe the instabilities.

Previous diagnostics by Andrews [1] using optical methods and Lagmuir probes showed that rotational oscillation modes existed in a steady state nitrogen plasma beam. It was decided to look for these oscillations using magnetic probes. Any such rotational characteristics of the plasma beam can be expected to produce sinusoidal fluctuations in the magnetic field containing the plasma. A Hewlett-Packard 3950A Wave analyzer was used to find the magnitudes and frequencies of the components present in the output of the magnetic probe. The output was in the form of a graph of component magnitude versus frequency. A typical graph is reproduced in Fig. 12.

The probe was moved radially and rotated to map the fields. At each position a graph was made. From each graph

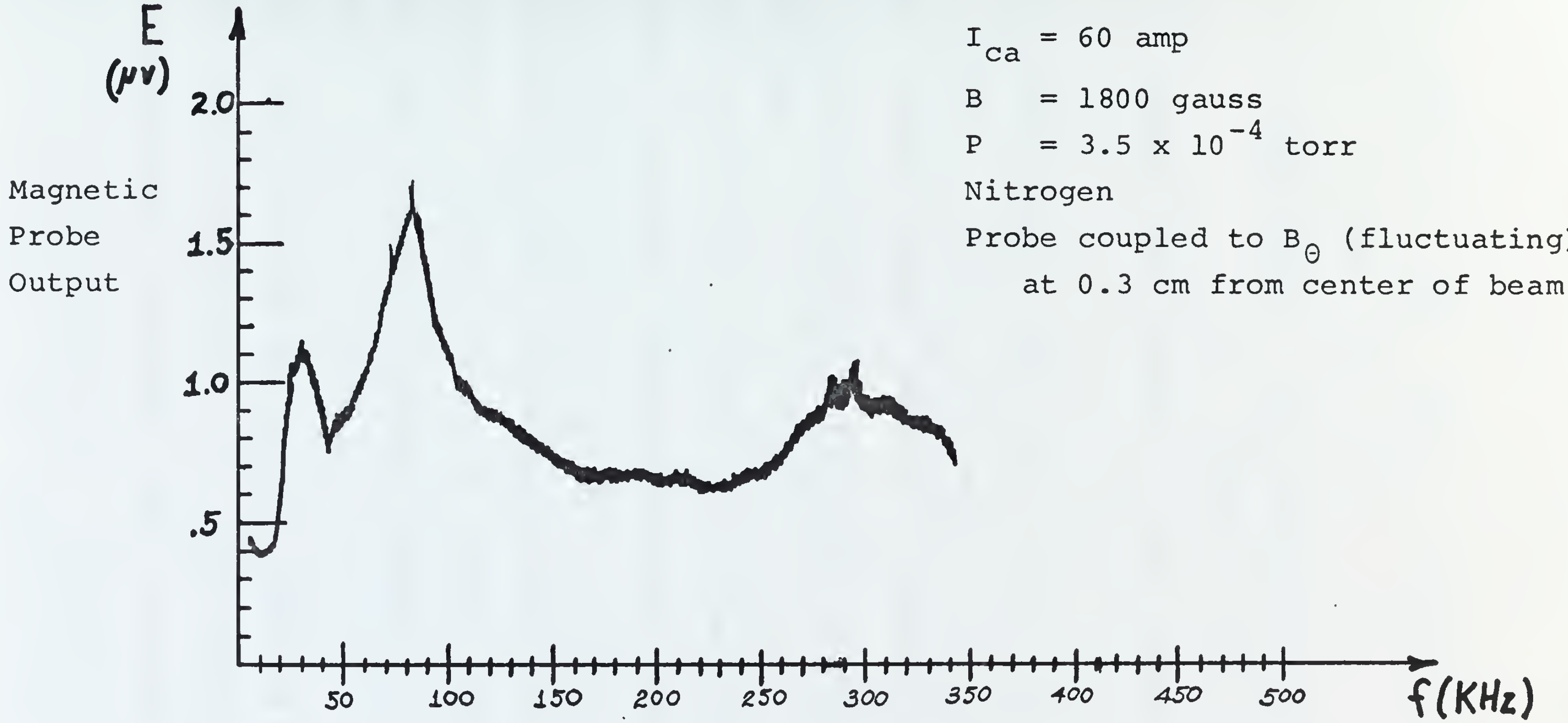


Figure 12. XY Recorder Trace from Wave Analyzer

the major frequencies were read, along with the magnitude of the probe output at each frequency. The probe output was then converted to B using the equation

$$B = \frac{1}{nA}\int e\,dt = \frac{E}{nA\omega} \quad \text{where } e = E \sin \omega t \ .$$

Since the output was in RMS,

$$B = \frac{1.414E\ (\text{RMS})}{nA\omega} \quad \text{for the peak of the sinusoidal magnetic field.}$$

Two graphs were obtained in this manner. (Figure 13.)

The largest fluctuation observed was a few guass at neutral gas pressure lower than in Fig. 12. The pinch will produce a field at the center of the pinch coil on the order of 20,000 gauss. Even with severe attenuation, shock waves should be detectable over the instabilities in the plasma beam.

As the probe was rotated at a given distance from the center of the beam a definite directionality of the fluctuating magnetic field was observed. It was determined that maximum probe output was obtained when the plane of the probe made an angle of about 15 degrees with the axis of the plasma beam. Thus the largest component of the fluctuating magnetic field is in the Theta direction. Andrews described a "skip rope" mode of oscillation observed with optical and Lagmuir probe diagnostics. For this mode the current is almost in the axial direction; thus the magnetic field is almost in the Theta direction. Therefore the directional nature of the magnetic field agrees with the current and

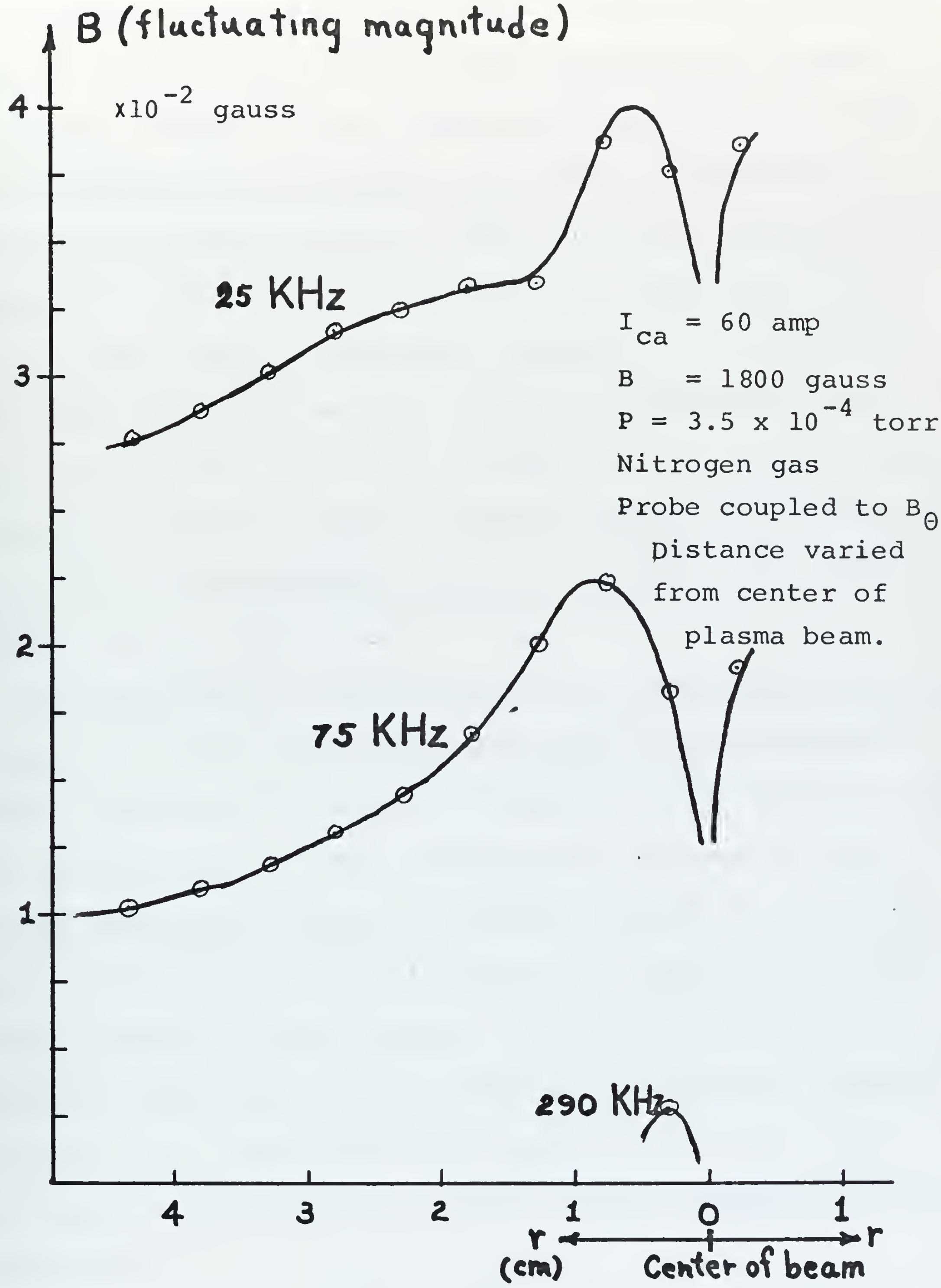


Figure 13. B_{Θ} Computed from Instability Probe Data

electric field requirements for Andrews' "skip rope" mode. The frequencies of oscillation are in close agreement with Andrews' "inner" and "outer" lobe observations of about 30 KHz and 100 KHz [1] at plasma conditions close to those used for Fig. 13. An attempt was made to resolve the direction of the fluctuating field by taking measurements of components parallel and perpendicular to the axis of the plasma beam. Little meaningful information was obtained. At this point the first magnetic probe was destroyed by the plasma after about ten hours of operation. A second magnetic probe similar in all ways to the first was constructed and prepared for measurements. It was destroyed within five minutes in the plasma beam.

The mechanism of destruction of the first probe appears to be slow in the sense that after each use it appeared a little darker with increasing charring of the internal parts. The second probe met with catastrophic failure in that it was burned completely through. The destruction of the second probe took place at the same time as a threefold increase in input pressure to the cathode. The sudden increase in pressure along with the fact that the cathode was close to the end of its life may offer some clue as to the reason for the failure. At any rate, future users of magnetic probes are warned!

The mechanism of destruction of the probes involves the energy density of the plasma and the rate at which that energy

is transferred to the probe. The rate of energy loss from the plasma is approximately the particle collision rate with the probe multiplied by the average kinetic energy per particle.

The particle flux to the probe is 1/4 $n\bar{v}$ and the average kinetic energy of each particle is 3/2kT. Thus the energy transfer to the probe per unit time per unit area of the probe surface will be

$$W = \frac{1}{4} n_e \bar{v}_e \left(\frac{3}{2} kT_e\right) + \frac{1}{4} n_i \bar{v}_i \left(\frac{3}{2} kT_i\right)$$

where n is the particle density, $\bar{v}$ is the average particle velocity, and T is the particle temperature. The subscripts e and i refer to electrons and ions respectively. Combining the equations yields

$$W = \frac{1}{4} n_e \sqrt{\frac{8kT_e}{\pi m_e}} \left(\frac{3}{2} kT_e\right) + \frac{1}{4} n_i \sqrt{\frac{8kT_i}{\pi m_i}} \left(\frac{3}{3} kT_i\right)$$

$$= \frac{3}{8} \sqrt{\frac{8k^3}{\pi}} \left[\frac{n_e T_e^{3/2}}{m_e^{1/2}} + \frac{n_i T_i^{3/2}}{m_i^{1/2}}\right]$$

For the nitrogen plasma $n_e = 2n_i$, $T_e >> T_i$ and $M_i >> M_e$, therefore the second term in the brackets is small compared with the first and will be neglected.

The energy transfer to the probe per unit area per unit time is

$$W = \frac{3}{8} n_e \left[\frac{8k^3 T_e^3}{\pi n}\right]^{1/2}$$

For the nitrogen plasma, using $n_e = 10^{19}\ m^{-3}$, $T_e = 4(10)^4\ {}^oK$, $m_e = 9.1(10)^{-31}$ kgm

$$W = \frac{3}{8}\,(10)^{19} \left[\frac{8\ (1.38)^3\ (10)^{-69}\ (4)^3\ (10)^{12}}{(3.14)(9.1)(10)^{-31}}\right]$$

$$= 7.3\ (10)^5\ \frac{\text{joules}}{\text{sec} - m^2} = 7.3\ (10)^5\ w/_{m}2$$

The part of the probe exposed to the dense part of the beam is a cylinder 3mm in diameter, 5mm long with a surface area of about $47 \times 10^{-6}\ m^2$.

The energy transferred to the probe per second is then

$$W_p = 7.3\ (10)^5\ (47)(10)^{-6} = 34.4w$$

W. Bötticher, the author of Chapter 2 of Ref. 2, solves the thermal diffusion equation for a probe and obtains an equation for the time at which the probe melts. His result is

$$t_m = \frac{1}{4}\pi c p \lambda\ \frac{T_m^{\ 2}}{q_o^{\ 2}} = \frac{Q}{q_o^{\ 2}}$$

where c = specific heat of probe material, p = density of probe material, λ = thermal conductivity of probe material, T_m = melting temperature of probe material, t_m = time at which probe material starts to melt, q_o = heat flux density, Q = grouping of constants.

For pyrex Q is approximately $10^4 \left[\frac{\text{joules}}{\text{sec cm}}\right]^2$ sec [2]

For the nitrogen plasma q_o = W (from above)

$$= 73\ \frac{\text{joules}}{\text{sec} - cm^2}$$

and $t_m = \dfrac{10^4}{(73)^2} = 1.9$ sec.

This approximation fails to consider many effects that might extend the life of the probe considerably such as the presence of a magnetic field and the charge sheath set up on the surface of the probe that excludes electrons with an energy less than the potential across the sheath. Therefore, an increase of several orders of magnitude in the time for the surface to melt is expected. The probe was moved every few minutes during the data taking process and did not spend long periods in the center of the plasma beam.

The first instability probe survived in a nitrogen beam with a neutral gas pressure of $3(10)^{-4}$ torr, cathode-to-anode current of 60 amps, and magnetic field of 1800 gauss. Therefore, it was able to withstand electron densities in the order of 10^{13} cm^{-3} with electron energies of 4ev. The second probe was destroyed when the density increased above this for a few minutes.

The problems of probe destruction are much different than those encountered with shock waves in which there is a change of parameters for a few microseconds. The energy transfer to the probe will be far less and the probes should survive in such a transient environment.

If further instability studies are made using magnetic probes, quartz or alumina could be used as the outer jacket. An alternative would be to construct a probe that could be

cooled. A double wall arrangement for the flow of cooling fluid is feasible, but the probe would probably have to be in the order of 2mm diameter at the tip to construct the probe and also to have a reasonable flow of cooling fluid.

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13. ABSTRACT

Inductive magnetic probes were studied, designed, constructed, calibrated and used at the Plasma Study Facility. The probes were built for three specific purposes: to measure current, radial shock waves and Alfvén waves in a fast Theta Pinch; to measure the parameters of shock fronts with short rise times produced by laser exploding of metal fibers; and to observe instabilities in a steady state plasma. Theta Pinch current measurements gave a current of 110 KAmps with the Theta Pinch operating with the capacitors charged to 13KV of 25KV maximum. The ringing frequency of the Theta Pinch was measured as 179 KHz. On calibration, the shock front probes showed a first resonance at 40 MHz which is comparable to probe characteristics found in recent literature. The instability probes gave information which agreed with previous descriptions of instabilities based on optical and Langmuir probe diagnostics.

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Plasma diagnostics						
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